

Research Article

A Note on the Generalized Nonlinear Vector Variational-Like Inequality Problem

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In this paper, we discuss two variants of the generalized nonlinear vector variational-like inequality problem. We provide their solutions by adopting topological approach. Topological properties such as compactness, closedness, and net theory are used in the proof. The admissibility of the function space topology and KKM-Theorem have played important role in proving the results.

1. Introduction

Variational inequalities have appeared as a working and important tool to investigate various fields of mathematics as well as of sciences including elasticity, vector equilibrium problems, and optimization problems [1-4]. In mid-sixties, Browder [5] formulated and proved the basic existence results for the solutions to a class of nonlinear variational inequality problems. He used a reflexive Banach space X and a monotone nonlinear map T from the space X to its dual space X^* , to set up the nonlinear variational inequality problem. Browder used the property of hemicontinuity and monotonicity of mapping T along with the lower semicontinuity of f, for providing the existence of the solution of nonlinear variational inequality problem. After that, this problem has been generalized and extended in various directions under different set-ups using different techniques. Liu et al. [6], Zhao et al. [7], and Ahmad and Irfan [8] are a few, who extended Browder's results to more generalized nonlinear variational inequalities. In 2009, Farajzadeh et al. [9] considered new kinds of generalized variational-like inequality problems under the frame work of topological vector spaces.

In the subsequent period, generalized quasi-variational inequalities were studied by Hung and others [10-12]. In 2017, Irfan et al. introduced a new generalized variationallike inclusion problem involving relaxed monotone operators [13]. A class of η -generalized operator variational-like inequalities were introduced by Kim et al. in 2018 [14]. In the same year, Tavakoli et al. studied the C-pseudomonotone property for the set-valued mappings in order to solve a generalized variational inequality problems [15]. On the other hand, vector equilibrium problems for the set-valued mappings were studied by Farajzadeh et al. and Chen et al. during this period [16, 17]. This wide range of literature is a clear indication of the importance that variational inequality problems have gained in the recent years. In this paper, we further add to this literature by providing solutions to a generalized nonlinear vector variational-like inequality problem, using topological methods.

Variational-like inequalities have number of applications which make it an interesting discipline for research. Vector variational inequality on flow equilibrium problem on a network has been discussed in [18]. Application of variational-like inequality in fuzzy optimization problem is discussed in [19]. More such studies are available in the literature [20–22].

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