

TOPOLOGICAL SOLUTIONS OF η -GENERALIZED VECTOR VARIATIONAL-LIKE INEQUALITY PROBLEMS

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Abstract. In this paper, we discuss several variants of the η -generalized vector variational-like inequality problem and provide existence theorems for their solutions via a topological approach. Several topological concepts like compactness, closedness, net theory and admissibility of function space topology are used for obtaining the main results. Finally, we give some topological properties of the solution set so obtained.

1. INTRODUCTION

In 1980, F. Gianessi [4] extended the concept of classical variational inequality (introduced by Stampacchia [23]) to vector variational inequality (VVI, in short) for vector valued functions in the setting of finite-dimensional Euclidean spaces. Further, VVI has been extended in various directions, in particular, the vector variational-like inequalities (VVLI, in short) [3,8,9,18,20]. VVI and their generalizations have been used extensively to solve vector optimization problems. Several researchers have established various relations between vector variational inequalities and vector optimization problems [12, 14, 24, 28].

In one direction, the concept of variational inequality was extended by Hanson [7] by introducing invex function (a generalization of convex function). Weir and Mond [25] and Noor [19] have studied some basic properties of preinvex and α -preinvex functions, respectively, along with their role in variational-like inequality problems and optimization problems. By assuming the condition of pseudoinvexity, Ruiz-Garzon et al. [20] have established some relations between vector variational-like inequality problems and optimization problems. In [8–10], Khan and others studied several variants of vector variational-like inequalities in the framework of Banach spaces. In 2017, Salahuddin [21] provided existence results for the solution of general set-valued vector variational inequalities. In the same year, Li and Yu [16] introduced a class of generalized invex functions, namely $(\alpha - \rho - \eta)$ -invex functions and provided the existence results for two types of vector variational-like inequalities. On the other hand, in 2018, Salahuddin [22] obtained the existence results for the solution of vector variational inequality problems by using sequentially continuous mapping. Recently, Gupta et al. [5] provided existence theorems for the solution of generalized non-linear vector variational-like

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