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A GENERALIZED APPROACH TOWARDS NORMALITY FOR TOPOLOGICAL SPACES

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Abstract. A uniform study towards normality is provided for topological spaces. Following Császár, γ-normality and $\gamma(\theta)$-normality are introduced and investigated. For $\gamma \in \Gamma_{13}$, γ-normality is found to satisfy Urysohn's lemma and provide partition of unity. Several existing variants of normality such as θ-normality, Δ-normality etc. are shown to be particular cases of $\gamma(\theta)$-normality. In this process, γ-regularity and $\gamma(\theta)$-regularity are introduced and studied. Several important characterizations of all these notions are provided.

1. Introduction

Normal spaces occupy a special place in the study of topological structures, as they are rich sources of continuous functions. Several important variants of normality are available in the literature such as θ-normality [7], Δ-normality [3] etc. Here arises a natural question: Does there exist a common approach to study these different variants of normality? In this paper, we have provided an answer to this question in affirmative. We have used monotonic mapping approach for this purpose. Following Császár [2], we have introduced γ-normality and $\gamma(\theta)$-normality for topological spaces. We have also introduced γ-regularity and $\gamma(\theta)$-regularity in the process. Some important characterizations of all these notions are provided. It is found that for $\gamma = int$, γ-normality and $\gamma(\theta)$-normality reduce to normality and θ-normality respectively. On the other hand, for $\gamma = cl\ int$, $\gamma(\theta)$-normality reduces to Δ-normality. Further it is proved that for $\gamma \in \Gamma_{13}$, γ-normal spaces satisfy Urysohn's lemma and provide partition of unity as well. Thus this study, while providing a uniform study of normality, has also brought to light a wide variety of topological spaces which are rich sources of continuous functions.

2. Basic Definitions and Preliminaries

Á. Császár [2] has used a map $\gamma : P(X) \longrightarrow P(X)$, where $P(X)$ is the power set of X, as his main tool for developing a generalized form of topological space. The

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