TEACHING PLAN for Academic Year 2021 – 2022

PAPER: Group Theory

SEMESTER: III

SESSION: 2021 – 2022 (Odd Semester)

TEACHER NAME: Dr. Ankit Gupta

- **SYLLABUS**

Unit 1: Groups and its Elementary Properties

Symmetries of a square, Dihedral groups, Definition and examples of groups including permutation groups and quaternion groups (illustration through matrices), Elementary properties of groups.

Unit 2: Subgroups and Cyclic Groups

Subgroups and examples of subgroups, Centralizer, Normalizer, Centre of a group, Product of two subgroups; Properties of cyclic groups, Classification of subgroups of cyclic groups.

Unit 3: Permutation Groups and Lagrange's Theorem

Cycle notation for permutations, Properties of permutations, Even and odd permutations, alternating groups; Properties of cosets, Lagrange's theorem and consequences including Fermat's Little theorem; Normal subgroups, Factor groups, Cauchy's theorem for finite abelian groups.

Unit 4: Group Homomorphisms

Group homomorphisms, Properties of homomorphisms, Group isomorphisms, Cayley's theorem, Properties of isomorphisms, First, Second and Third isomorphism theorems for groups.

- **COURSE DESCRIPTION/OBJECTIVE**

The objective of the course is to introduce the fundamental theory of groups and their homomorphisms. Symmetric groups and group of symmetries are also studied in detail. Fermat's Little theorem as a consequence of the Lagrange's theorem on finite groups.

- **TEACHING TIME (No. Of Weeks): 14 weeks (Approximately)**

- **CLASSES**

The course is organized around daily lectures as per the time table scheduled in online mode via Google Meet. Students will be given readings each week to help them follow the course content. These readings will be discussed in class in detail.

Recordings of the lectures were also provided on daily basis.

WEEK WISE BREAK UP OF SYLLABUS

Week 1: Symmetries of a square, Dihedral groups, Definition and examples of groups including permutation groups and quaternion groups (illustration through matrices).
[1] Chapter 1.

Week 2: Definition and examples of groups, Elementary properties of groups.
[1] Chapter 2.

Week 3: Subgroups and examples of subgroups, Centralizer, Normalizer, Center of a Group, Product of two subgroups.
[1] Chapter 3.

Weeks 4 and 5: Properties of cyclic groups. Classification of subgroups of cyclic groups.
[1] Chapter 4

Weeks 6 and 7: Cycle notation for permutations, Properties of permutations, Even and odd permutations, Alternating group.
[1] Chapter 5 (up to Page 110).

Weeks 8 and 9: Properties of cosets, Lagrange's theorem and consequences including Fermat's Little theorem.
[1] Chapter 7 (up to Example 6, Page 150).

Week 10: Normal subgroups, Factor groups, Cauchy's theorem for finite abelian groups.
[1] Chapters 9 (Theorem 9.1, 9.2, 9.3 and 9.5, and Examples 1 to 12).

Weeks 11 and 12: Group homomorphisms, Properties of homomorphisms, Group isomorphisms, Cayley's theorem.
[1] Chapter 10 (Theorems 10.1 and 10.2, Examples 1 to 11).
[1] Chapter 6 (Theorem 6.1, and Examples 1 to 8).

Weeks 13 and 14: Properties of isomorphisms, First, Second and Third isomorphism theorems.
[1] Chapter 6 (Theorems 6.2 and 6.3), Chapter 10 (Theorems 10.3, 10.4, Examples 12 to 14, and Exercises 41 and 42 for second and third isomorphism theorems for groups).

- **ASSESSMENT**

Internal Assessment: 25 Marks (Group Assignment, presentation, Class Test and Attendance)

- **ESSENTIAL READINGS**

[1] Gallian, Joseph. A. (2013). Contemporary Abstract Algebra (8th ed.). Cengage Learning India Private Limited, Delhi. Fourth impression, 2015.

- **SUGGESTED READINGS**

[1] Rotman, Joseph J. (1995). An Introduction to The Theory of Groups (4th ed.). Springer-Verlag, New York.

TEACHING PLAN for Academic Year 2021 - 2022

PAPER: Metric Spaces
SEMESTER: V
SESSION: 2021 – 2022 (Odd Semester)
TEACHER NAME: Dr. Ankit Gupta

- **SYLLABUS**

Unit 1: Basic Concepts
Metric spaces**:** Definition and examples, Sequences in metric spaces, Cauchy sequences, Complete metric space.

Unit 2: Topology of Metric Spaces
Open and closed ball, Neighborhood, Open set, Interior of a set, Limit point of a set, Derived set, Closed set, Closure of a set, Diameter of a set, Cantor's theorem, Subspaces, Dense set.

Unit 3: Continuity & Uniform Continuity in Metric Spaces
Continuous mappings, Sequential criterion and other characterizations of continuity, Uniform continuity, Homeomorphism, Contraction mapping, Banach fixed point theorem.

Unit 4: Connectedness and Compactness
Connectedness, Connected subsets of $\mathbb{R}$, Connectedness and continuous mappings, Compactness, Compactness and boundedness, Continuous functions on compact spaces.

- **COURSE DESCRIPTION/OBJECTIVE**

Up to this stage, students do study the concepts of analysis which evidently rely on the notion of distance. In this course, the objective is to develop the usual idea of distance into an abstract form on any set of objects, maintaining its inherent characteristics, and the resulting consequences.

- **TEACHING TIME(No. Of Weeks): 14 weeks (Approximately)**

- **CLASSES**

The course is organized around daily lectures as per the time table scheduled via online mode. Students will be given readings and other e-materials each week to help them follow the course content. These materials will be discussed in class in detail.

- **WEEK WISE BREAK UP OF SYLLABUS**

Week 1: Definition of metric space, Illustration using the usual metric on $\mathbb{R}$, Euclidean and max metric on $\mathbb{R}$C, Euclidean and max metric on $\mathbb{R}$_, Discrete metric, Sup metric on B(S) and C[a, b], Integral metric on C[a, b].
[1] Chapter 1 [Section 1.2 (1.2.1, 1.2.2 ((i), (ii), (iv), (v), (viii), (ix), (x)), 1.2.3 and 1.2.4 (i))]

Week 2: Sequences in metric space, Definition of limit of a sequence, Illustration through examples, Cauchy sequences.
[1] Chapter 1 [Section 1.3 (1.3.1, 1.3.2, 1.3.3 ((i), (iv)), 1.3.5) and Section 1.4 (1.4.1 to 1.4.4)]

Week 3: Definition of complete metric spaces, Illustration through examples.
[1] Chapter 1 [Section 1.4 (1.4.5 to 1.4.7, 1.4.12 to 1.4.14(ii))].

Week 4: Open and closed balls, Neighborhood, Open sets, Examples and basic results.
[1] Chapter 2 [Section 2.1 (2.1.1 to 2.1.11 (except 2.1.6(ii)))].

Week 5: Interior point, Interior of a set, Limit point, Derived set, Examples and basic results.
[1] Chapter 2 [Section 2.1 (2.1.12 to 2.1.20)].

Week 6: Closed set, Closure of a set, Examples and basic results.
[1] Chapter 2 [Section 2.1 (2.1.21 to 2.1.35)].

Week 7: Bounded set, Diameter of a set, Cantor's theorem.
[1] Chapter 2 [Section 2.1 (2.1.41 to 2.1.44)].

Week 8: Relativisation and subspaces, Dense sets.
[1] Chapter 2 [Section 2.2 (2.2.1 to 2.2.6), Section 2.3 (2.3.12 to 2.3.13(iv))].

Weeks 9 to 11: Continuous mappings, Sequential and other characterizations of continuity, Uniform continuity, Homeomorphism, Contraction mappings, Banach fixed point theorem.
[1] Chapter 3 [Section 3.1, Section 3.4 (3.4.1 to 3.4.8), Section 3.5 (3.5.1 to 3.5.7(iii)), and
Section 3.7 (3.7.1 to 3.7.5)].

Weeks 12 to 14: Connectedness and compactness, Definitions and properties of connected and compact spaces.
[1] Chapter 4 [Section 4.1 (4.1.1 to 4.1.12)], and Chapter 5 [Section 5.1 (5.1.1 to 5.1.6), and
Section 5.3 (5.3.1 to 5.3.10)].

- **ASSESSMENT**

Internal Assessment: 25 Marks (Group Assignment, presentation, Class Test and Attendance)

- **ESSENTIAL READINGS**

[1] Shirali, Satish & Vasudeva, H. L. (2009). *Metric Spaces*, Springer, First Indian Print.

- **SUGGESTED READINGS**

[1] Kumaresan, S. (2014). *Topology of Metric Spaces* (2nd ed.). Narosa Publishing House. New Delhi.
[2] Simmons, George F. (2004). *Introduction to Topology and Modern Analysis*. McGraw-Hill Education. New Delhi.
