PAPER: SEC – 2: Computer Algebra Systems and Related Software SEMESTER: IV SESSION: 2020 – 2021 (Even Semester) TEACHER NAME: Dr. Ankit Gupta

• SYLLABUS

Unit 1: Introduction to CAS and Applications

Computer Algebra System (CAS), Use of a CAS as a calculator, Computing and plotting functions in 2D, Plotting functions of two variables using Plot3D and Contour Plot, Plotting parametric curves surfaces, Customizing plots, Animating plots, Producing tables of values, working with piecewise defined functions, Combining graphics.

Unit 2: Working with Matrices

Simple programming in a CAS, Working with matrices, Performing Gauss elimination, operations (transpose, determinant, inverse), Minors and cofactors, Working with large matrices, Solving system of linear equations, Rank and nullity of a matrix, Eigenvalue, eigenvector and diagonalization.

Unit 3: R - The Statistical Programming Language

R as a calculator, Explore data and relationships in **R**. Reading and getting data into **R**: Combine and scan commands, Types and structure of data items with their properties, Manipulating vectors, Data frames, Matrices and lists, Viewing objects within objects, Constructing data objects and conversions.

Unit 4: Data Analysis with R

Summary commands: Summary statistics for vectors, Data frames, Matrices and lists, Summary tables, Stem and leaf plot, Histograms, Plotting in **R**: Box-whisker plots, Scatter plots, Pairs plots, Line charts, Pie charts, Cleveland dot charts and bar charts, Copy and save graphics to other applications.

• COURSE DESCRIPTION/OBJECTIVE

This course aims at familiarizing students with the usage of computer algebra systems (Mathematica and Maxima) and the statistical software \mathbf{R} . The basic emphasis is on plotting and working with matrices using CAS. Data entry and summary commands will be studied in \mathbf{R} . Graphical representation of data shall also be explored.

• **TEACHING TIME(No. Of Weeks): 14 weeks (Approximately)**

• CLASSES

The course is organized around daily lectures as per the time table scheduled on online mode. Students will be given readings along with other e-materials each week to help them follow the course content. These readings will be discussed in class in detail.

• WEEK WISE BREAK UP OF SYLLABUS

Weeks 1 to 3: Computer Algebra System (CAS), Use of a CAS as a calculator, Computing and plotting functions in 2D, Producing tables of values, Working with piecewise defined functions,

Combining graphics. Simple programming in a CAS.

[1] Chapter 12 (Sections 12.1 to 12.5).

[2] Chapter 1, and Chapter 3 (Sections 3.1 to 3.6 and 3.8).

Weeks 4 and 5: Plotting functions of two variables using Plot3D and contour plot, Plotting

parametric curves surfaces, Customizing plots, Animating plots. [2] Chapter 6 (Sections 6.2 and 6.3).

Weeks 6 to 8: Working with matrices, Performing Gauss elimination, Operations (Transpose, Determinant, Inverse), Minors and cofactors, Working with large matrices, Solving system of linear equations, Rank and nullity of a matrix, Eigenvalue, Eigenvector and diagonalization.

[2] Chapter 7 (Sections 7.1 to 7.8).

Weeks 9 to 11: **R** as a calculator, Explore data and relationships in **R**. Reading and getting data into **R**: Combine and scan commands, Types and structure of data items with their properties. Manipulating vectors, Data frames, Matrices and lists. Viewing objects within objects. Constructing data objects and conversions.

[1] Chapter 14 (Sections 14.1 to 14.4).

[3] Chapter 2, and Chapter 3.

Weeks 12 to 14: Summary commands: Summary statistics for vectors, Data frames, Matrices and lists. Summary tables. Stem and leaf plot, histograms. Plotting in R: Box-whisker plots, Scatter plots, Pairs plots, Line charts, Pie charts, Cleveland dot charts and Bar charts. Copy and save graphics to other applications.

[1] Chapter 14 (Section 14.7).

[3] Chapter 5 (up to Page 157), and Chapter 7.

• ASSESSMENT

Internal Assessment: 25 Marks (Group Assignment, presentation and Attendance)

• ESSENTIAL READINGS

- [1] Bindner, Donald & Erickson, Martin. (2011). A Student's Guide to the Study, Practice, and Tools of Modern Mathematics. CRC Press, Taylor & Francis Group, LLC.
- [2] Torrence, Bruce F., & Torrence, Eve A. (2009). *The Student's Introduction to Mathematica: A Handbook for Precalculus, Calculus, and Linear Algebra* (2nd ed.). Cambridge University Press.
- [3] Gardener, M. (2012). Beginning R: The Statistical Programming Language, Wiley.

• SUGGESTED READINGS

[1] Verzani, John (2014). *Using R for Introductory Statistics* (2nd ed.). CRC Press, Taylor & Francis Group.

PAPER: Group Theory SEMESTER: III SESSION: 2020 – 2021 (Odd Semester) TEACHER NAME: Dr. Ankit Gupta

• SYLLABUS

Unit 1: Groups and its Elementary Properties

Symmetries of a square, Dihedral groups, Definition and examples of groups including permutation groups and quaternion groups (illustration through matrices), Elementary properties of groups.

Unit 2: Subgroups and Cyclic Groups

Subgroups and examples of subgroups, Centralizer, Normalizer, Centre of a group, Product of two subgroups; Properties of cyclic groups, Classification of subgroups of cyclic groups.

Unit 3: Permutation Groups and Lagrange's Theorem

Cycle notation for permutations, Properties of permutations, Even and odd permutations, alternating groups; Properties of cosets, Lagrange's theorem and consequences including Fermat's Little theorem; Normal subgroups, Factor groups, Cauchy's theorem for finite abelian groups.

Unit 4: Group Homomorphisms

Group homomorphisms, Properties of homomorphisms, Group isomorphisms, Cayley's theorem, Properties of isomorphisms, First, Second and Third isomorphism theorems for groups.

• COURSE DESCRIPTION/OBJECTIVE

The objective of the course is to introduce the fundamental theory of groups and their homomorphisms. Symmetric groups and group of symmetries are also studied in detail. Fermat's Little theorem as a consequence of the Lagrange's theorem on finite groups.

- TEACHING TIME (No. Of Weeks): 14 weeks (Approximately)
- CLASSES

The course is organized around daily lectures as per the time table scheduled in online mode via Google Meet. Students will be given readings each week to help them follow the course content. These readings will be discussed in class in detail.

Recordings of the lectures were also provided on daily basis.

WEEK WISE BREAK UP OF SYLLABUS

Week 1: Symmetries of a square, Dihedral groups, Definition and examples of groups including permutation groups and quaternion groups (illustration through matrices). [1] Chapter 1.

Week 2: Definition and examples of groups, Elementary properties of groups. [1] Chapter 2.

Week 3: Subgroups and examples of subgroups, Centralizer, Normalizer, Center of a Group, Product of two subgroups. [1] Chapter 3.

Weeks 4 and 5: Properties of cyclic groups. Classification of subgroups of cyclic groups.

[1] Chapter 4

Weeks 6 and 7: Cycle notation for permutations, Properties of permutations, Even and odd permutations, Alternating group. [1] Chapter 5 (up to Page 110).

Weeks 8 and 9: Properties of cosets, Lagrange's theorem and consequences including Fermat's Little theorem.

[1] Chapter 7 (up to Example 6, Page 150).

Week 10: Normal subgroups, Factor groups, Cauchy's theorem for finite abelian groups.

[1] Chapters 9 (Theorem 9.1, 9.2, 9.3 and 9.5, and Examples 1 to 12).

Weeks 11 and 12: Group homomorphisms, Properties of homomorphisms, Group isomorphisms, Cayley's theorem.

[1] Chapter 10 (Theorems 10.1 and 10.2, Examples 1 to 11).

[1] Chapter 6 (Theorem 6.1, and Examples 1 to 8).

Weeks 13 and 14: Properties of isomorphisms, First, Second and Third isomorphism theorems.

[1] Chapter 6 (Theorems 6.2 and 6.3), Chapter 10 (Theorems 10.3, 10.4, Examples 12 to 14, and Exercises 41 and 42 for second and third isomorphism theorems for groups).

• ASSESSMENT

Internal Assessment: 25 Marks (Group Assignment, presentation, Class Test and Attendance)

• ESSENTIAL READINGS

[1] Gallian, Joseph. A. (2013). Contemporary Abstract Algebra (8th ed.). Cengage Learning India Private Limited, Delhi. Fourth impression, 2015.

• SUGGESTED READINGS

 Rotman, Joseph J. (1995). An Introduction to The Theory of Groups (4th ed.). Springer-Verlag, New York.

PAPER: Metric Spaces SEMESTER: V SESSION: 2020 – 2021 (Odd Semester) TEACHER NAME: Dr. Ankit Gupta

• SYLLABUS

Unit 1: Basic Concepts

Metric spaces: Definition and examples, Sequences in metric spaces, Cauchy sequences, Complete metric space.

Unit 2: Topology of Metric Spaces

Open and closed ball, Neighborhood, Open set, Interior of a set, Limit point of a set, Derived set, Closed set, Closure of a set, Diameter of a set, Cantor's theorem, Subspaces, Dense set.

Unit 3: Continuity & Uniform Continuity in Metric Spaces

Continuous mappings, Sequential criterion and other characterizations of continuity, Uniform continuity, Homeomorphism, Contraction mapping, Banach fixed point theorem.

Unit 4: Connectedness and Compactness

Connectedness, Connected subsets of \mathbb{R} , Connectedness and continuous mappings, Compactness, Compactness and boundedness, Continuous functions on compact spaces.

• COURSE DESCRIPTION/OBJECTIVE

Up to this stage, students do study the concepts of analysis which evidently rely on the notion of distance. In this course, the objective is to develop the usual idea of distance into an abstract form on any set of objects, maintaining its inherent characteristics, and the resulting consequences.

• **TEACHING TIME**(No. Of Weeks): 14 weeks (Approximately)

• CLASSES

The course is organized around daily lectures as per the time table scheduled via online mode. Students will be given readings and other e-materials each week to help them follow the course content. These materials will be discussed in class in detail.

• WEEK WISE BREAK UP OF SYLLABUS

Week 1: Definition of metric space, Illustration using the usual metric on \mathbb{R} , Euclidean and max metric on \mathbb{R} C, Euclidean and max metric on \mathbb{R}_{-} , Discrete metric, Sup metric on B(*S*) and C[*a*, *b*], Integral metric on C[*a*, *b*].

[1] Chapter 1 [Section 1.2 (1.2.1, 1.2.2 ((i), (ii), (iv), (v), (viii), (ix), (x)), 1.2.3 and 1.2.4 (i))]

Week 2: Sequences in metric space, Definition of limit of a sequence, Illustration through examples, Cauchy sequences.

[1] Chapter 1 [Section 1.3 (1.3.1, 1.3.2, 1.3.3 ((i), (iv)), 1.3.5) and Section 1.4 (1.4.1 to 1.4.4)]

Week 3: Definition of complete metric spaces, Illustration through examples. [1] Chapter 1 [Section 1.4 (1.4.5 to 1.4.7, 1.4.12 to 1.4.14(ii))].

Week 4: Open and closed balls, Neighborhood, Open sets, Examples and basic results.

[1] Chapter 2 [Section 2.1 (2.1.1 to 2.1.11 (except 2.1.6(ii)))].

Week 5: Interior point, Interior of a set, Limit point, Derived set, Examples and basic results.

[1] Chapter 2 [Section 2.1 (2.1.12 to 2.1.20)].

Week 6: Closed set, Closure of a set, Examples and basic results. [1] Chapter 2 [Section 2.1 (2.1.21 to 2.1.35)].

Week 7: Bounded set, Diameter of a set, Cantor's theorem. [1] Chapter 2 [Section 2.1 (2.1.41 to 2.1.44)].

Week 8: Relativisation and subspaces, Dense sets. [1] Chapter 2 [Section 2.2 (2.2.1 to 2.2.6), Section 2.3 (2.3.12 to 2.3.13(iv))].

Weeks 9 to 11: Continuous mappings, Sequential and other characterizations of continuity, Uniform continuity, Homeomorphism, Contraction mappings, Banach fixed point theorem.

[1] Chapter 3 [Section 3.1, Section 3.4 (3.4.1 to 3.4.8), Section 3.5 (3.5.1 to 3.5.7(iii)), and

Section 3.7 (3.7.1 to 3.7.5)].

Weeks 12 to 14: Connectedness and compactness, Definitions and properties of connected and compact spaces.

[1] Chapter 4 [Section 4.1 (4.1.1 to 4.1.12)], and Chapter 5 [Section 5.1 (5.1.1 to 5.1.6), and Section 5.3 (5.3.1 to 5.3.10)].

• ASSESSMENT

Internal Assessment: 25 Marks (Group Assignment, presentation, Class Test and Attendance)

ESSENTIAL READINGS
[1] Shirali, Satish & Vasudeva, H. L. (2009). *Metric Spaces*, Springer, First Indian Print.

• SUGGESTED READINGS

[1] Kumaresan, S. (2014). *Topology of Metric Spaces* (2nd ed.). Narosa Publishing House. New Delhi.

[2] Simmons, George F. (2004). *Introduction to Topology and Modern Analysis*. McGraw-Hill Education. New Delhi.

PAPER: Complex Analysis SEMESTER: VI SESSION: 2020 – 2021 (Even Semester) TEACHER NAME: Dr. Ankit Gupta

• SYLLABUS

Unit 1: Analytic Functions and Cauchy - Riemann Equations

Functions of complex variable, Mappings; Mappings by the exponential function, Limits, Theorems on limits, Limits involving the point at infinity, Continuity, Derivatives, Differentiation formulae, Cauchy Riemann equations, Sufficient conditions for differentiability; Analytic functions and their examples.

Unit 2: Elementary Functions and Integrals

Exponential function, Logarithmic function, Branches and derivatives of logarithms, Trigonometric function, Derivatives of functions, Definite integrals of functions, Contours, Contour integrals and its examples, Upper bounds for moduli of contour integrals.

Unit 3: Cauchy's Theorems and Fundamental Theorem of Algebra

Antiderivatives, Proof of antiderivative theorem, Cauchy - Goursat theorem, Cauchy integral formula; An extension of Cauchy integral formula, Consequences of Cauchy integral formula, Liouville's theorem and the fundamental theorem of algebra.

Unit 4: Series and Residues

Convergence of sequences and series, Taylor series and its examples; Laurent series and its examples, Absolute and uniform convergence of power series, Uniqueness of series representations of power series, Isolated singular points, Residues, Cauchy's residue theorem, residue at infinity; Types of isolated singular points, Residues at poles and its examples.

• COURSE DESCRIPTION/OBJECTIVE

This course aims to introduce the basic ideas of analysis for complex functions in complex variables with visualization through relevant practicals. Emphasis has been laid on Cauchy's theorems, series expansions and calculation of residues.

• **TEACHING TIME(No. Of Weeks): 14 weeks (Approximately)**

• CLASSES

The course is organized around daily lectures as per the time table scheduled on online mode. Students will be given readings along with other e-materials each week to help them follow the course content. These readings will be discussed in class in detail.

• WEEK WISE BREAK UP OF SYLLABUS

Week 1: Functions of complex variable, Mappings, Mappings by the exponential function.

[1] Chapter 2 (Sections 12 to 14).

Week 2: Limits, Theorems on limits, Limits involving the point at infinity, Continuity. [1] Chapter 2 (Sections 15 to 18).

Week 3: Derivatives, Differentiation formulae, Cauchy-Riemann equations, Sufficient conditions for differentiability. [1] Chapter 2 (Sections 19 to 22).

Week 4: Analytic functions, Examples of analytic functions, Exponential function. [1] Chapter 2 (Sections 24 and 25) and Chapter 3 (Section 29).

Week 5: Logarithmic function, Branches and Derivatives of Logarithms, Trigonometric functions.[1] Chapter 3 (Sections 30, 31 and 34).

Week 6: Derivatives of functions, Definite integrals of functions, Contours. [1] Chapter 4 (Sections 37 to 39).

Week 7: Contour integrals and its examples, upper bounds for moduli of contour integrals. [1] Chapter 4 (Sections 40, 41 and 43).

Week 8: Antiderivatives, proof of antiderivative theorem. [1] Chapter 4 (Sections 44 and 45).

Week 9: State Cauchy- Goursat theorem, Cauchy integral formula. [1] Chapter 4 (Sections 46 and 50).

Week 10: An extension of Cauchy integral formula, Consequences of Cauchy integral formula,

Liouville's theorem and the fundamental theorem of algebra. [1] Chapter 4 (Sections 51 to 53).

Week 11: Convergence of sequences, Convergence of series, Taylor series, proof of Taylor's theorem, Examples. [1] Chapter 5 (Sections 55 to 59).

Week 12: Laurent series and its examples, Absolute and uniform convergence of power series, uniqueness of series representations of power series. [1] Chapter 5 (Sections 60, 62, 63 and 66).

Week 13: Isolated singular points, Residues, Cauchy's residue theorem, Residue at infinity. [1]: Chapter 6 (Sections 68 to 71).

Week 14: Types of isolated singular points, Residues at poles and its examples. [1] Chapter 6 (Sections 72 to 74).

• ASSESSMENT

Internal Assessment: 25 Marks (Group Assignment, presentation, Class Test and Attendance)

• ESSENTIAL READINGS

[1] Brown, James Ward, & Churchill, Ruel V. (2014). *Complex Variables and Applications* (9th ed.). McGraw-Hill Education. New York.

• SUGGESTED READINGS

- [1] Bak, Joseph & Newman, Donald J. (2010). *Complex Analysis* (3rd ed.). Undergraduate Texts in Mathematics, Springer. New York.
- [2] Zills, Dennis G., & Shanahan, Patrick D. (2003). *A First Course in Complex Analysis with Applications*. Jones & Bartlett Publishers, Inc.
- [3] Mathews, John H., & Howell, Rusell W. (2012). Complex Analysis for Mathematics and Engineering (6th ed.). Jones & Bartlett Learning. Narosa, Delhi. Indian Edition.
